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VIBRATORY BEHAVIOR CALCULATION OF A ELECTRIC MOTOR APPLYING A 3D STRUCTURAL MODEL

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Abstract. *In this paper a simplified approach is introduced to model forced vibrations of magnetic origin in non-skewed electric machines. In this approach a 2D finite element electromagnetic model is used to obtain the magnetic forces and a 3D structural finite element model is used to calculate the vibrations. The forced vibrations calculations are validated by measurements. This procedure is based on the assumption that the tangential and radial components of the magnetic forces are constant through the longitudinal axis in non-skewed electric machines.*

Keywords: *Vibration, Magnetic forces, Maxwell's stress tensor, Finite element method, Switched reluctance motor.*

1. INTRODUCTION

In this work a simplified procedure is introduced to model forced vibrations of magnetic origin in non-skewed electric machines**.**

Two dimensional finite elements are used to evaluate the electromagnetic forces. With this procedure, valid for non - skewed electrical machines as the Switched Reluctance Motor (Neves *et all.*, 1997) used in this investigation, the tangential and radial components of the magnetic forces are constant through the longitudinal axis direction.

In the mechanical point of view the stiffness changes through the longitudinal axis due to geometric characteristics of motor structure affecting the vibrations. So, consistent results can only be obtained if a 3D finite element structural mesh, supposing that they act at equidistant plans through the axis direction.

With the forces and the 3D mechanical model, the forced vibrations are obtained using the Modal Superposition Method in which the response of a continuos structure to any force can be represented by the superposition of the various responses in their individual modes, considering each mode to respond as a single degree freedom system. This method requires a natural response calculation prior to further solution steps.

The modal damping coefficients measured by experimental modal analysis are used as input variables in the forced response calculation.

This simplified approach is applied to simulate the vibrations of magnetic origin in a commercial 8/6 poles Switched Reluctance Motor (SRM).

The flow chart of "Fig. 1" summarizes the simplified approach.

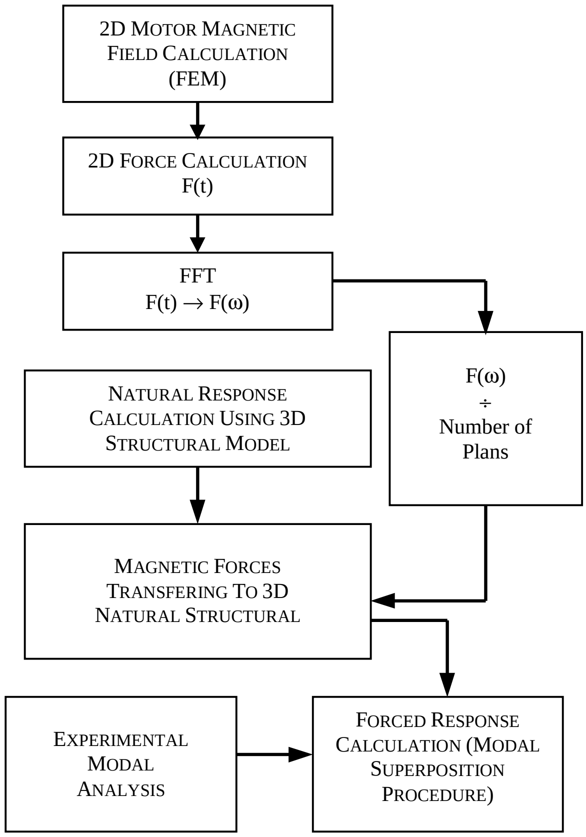

Figure 1 - Flow chart describing the simplified approach.

2. MAGNETIC FORCE CALCULATION

The windings currents of a SRM are unidirectional, and so, the frequency of the fundamental of the magnetic force over a stator pole is the same as that current. Nevertheless, due to the modification of the inductance with rotation and also due to the modification of the currents waveforms with changes in the switching angles, analytical calculation of the radial forces acting on the stator teeth is a difficult task. On the other hand, Maxwell Stress Tensor applied to field distribution obtained with Finite Element Method is a practical way to determine the radial magnetic forces on a tooth (Sadowski *et all*., 1992).

The Maxwell Stress Tensor, applied on a surface in the air gap gives a magnetic force density which can be written as:

$$\frac{d\mathbf{f}}{ds} = \frac{1}{\mu_0}\left[(\mathbf{n}\cdot\mathbf{B})\mathbf{B} - \frac{1}{2}B^2\mathbf{n}\right] \qquad (1)$$

where μ_0 is the permeability of the air, **n** is the vector normal to iron and **B** is the air side magnetic induction obtained with 2D Finite Element calculations. In order to simplify the analysis, for each rotor position, the force distribution on a tooth is integrated and then concentrated on a point in the center of the inner surface of the tooth.

The calculated radial force acting on a tooth as a function of time and at speed of 2500 rpm is shown in “Fig. 2(a)” and the corresponding harmonic spectrum obtained using Fourier analysis is shown in “Fig. 2(b)”.

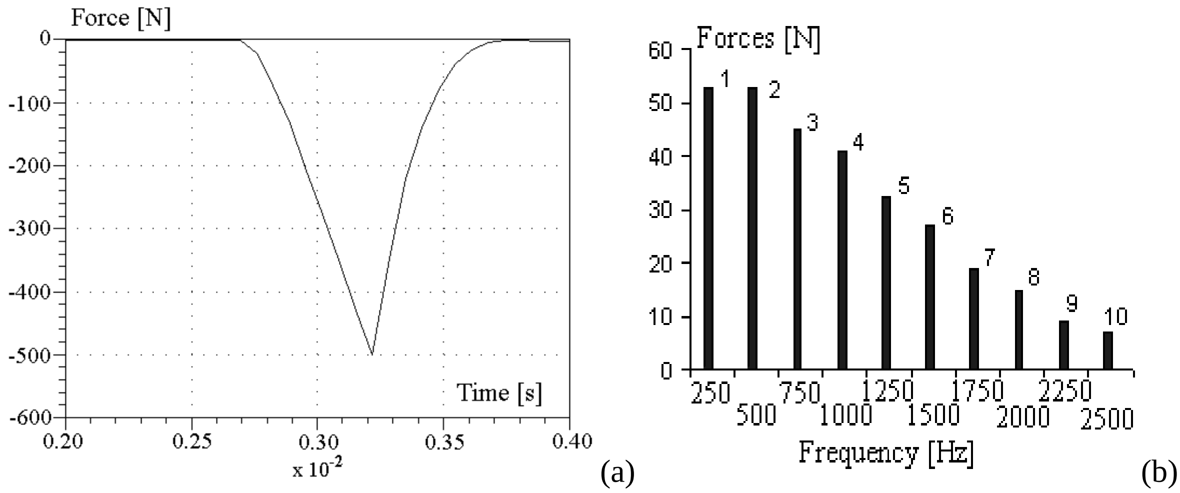

Figure 2 - Radial magnetic forces. (a) as a function of time. (b) as a function of frequency.

3. NATURAL RESPONSE CALCULATION

The natural response is important as a precursor to any dynamic analysis because knowledge of the structure's fundamental modes and frequencies can help to characterize its dynamic response. Additionally, some forced response solution procedures, as the modal superposition method, requires the results of a natural response calculation.

The natural frequencies and corresponding vibration modes can be obtained, using structural Finite Elements Method, by solving (2):

$$\mathbf{Ku}_r = \lambda \mathbf{Mu}_r \tag{2}$$

where λ is an eigenvalue; $\mathbf{u_r}$ is the corresponding eigenvector; **K** and **M** are, respectively, the rigidity and the mass matrices. Mechanical damping is neglected.

Figure 3 shows the finite element mesh composed of 53040 elements and 14229 nodes. Figures 4 and 5 show, respectively, the SRM structure mode shapes for the natural frequencies of 1377 Hz and 1773 Hz, obtained by the ANSYS mechanical software.

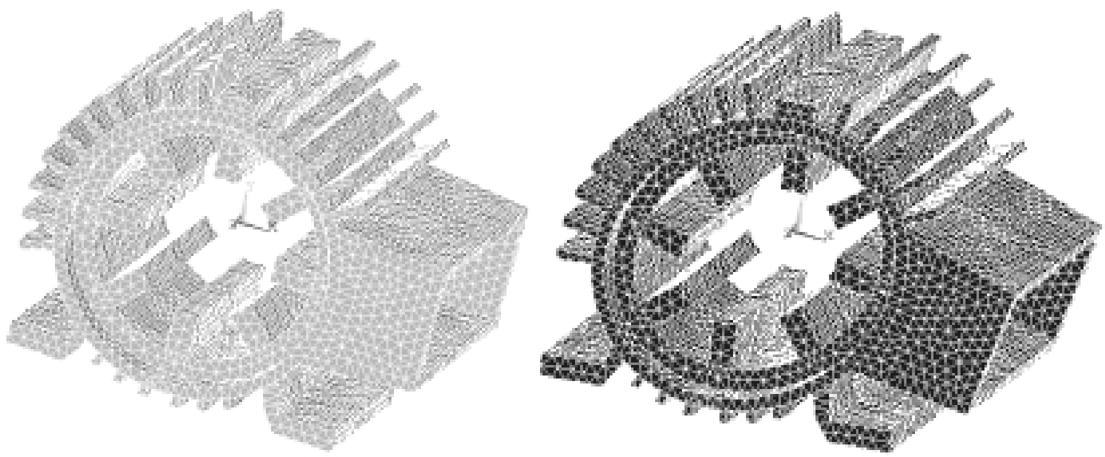

Figure3 - (a) Three dimensional finite element structural mesh. (b) Calculated structural mode shape for the 1377 Hz natural frequency.

4. MAGNETIC FORCES TRANSFERING TO 3D NATURAL STRUCTURAL MESH

The magnetic forces vectors decomposed in Cartesian components for each harmonic are applied to the natural structural mesh at nodes located in the center of the inner surface of each stator tooth. This procedure are repeated for each chosen 3D structural mesh equidistant axial plane. The force vectors magnitudes are divided by the number of plans.

Figure 4 shows the 5th harmonic magnetic force vectors applied to the SRM structure at ten equidistant plans.

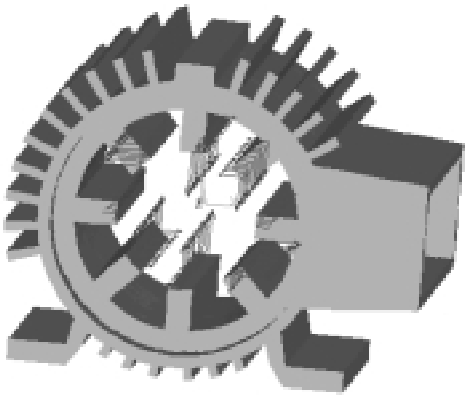

Figure 4 - 5th harmonic magnetic force vectors applied to the SRM structure.

5. MECHANICAL FORCED RESPONSE - MODAL SUPERPOSITION METHOD

Using the Modal Superposition Method the forced vibration response of a continuous structure (i. e. multiple degree of freedom system) to any force can be represented by the superposition of the various responses in their individual modes, considering each mode to respond as single degree freedom system. This method requires a natural response calculation prior to further solution steps.

The general equation of motion can be written as:

$$\mathbf{M}\frac{d^2}{dt^2}\mathbf{q}(t)+\mathbf{C}\frac{d}{dt}\mathbf{q}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{F}(t) \quad (3)$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are respectively the mass, damping and rigidity matrices, $\mathbf{q}(t)$ is the displacement vector and $\mathbf{F}(t)$ is the force vector. Using the next transformation matrix:

$$\mathbf{q}(t)=\mathbf{U}\,\mathbf{p}(t) \quad \text{or} \quad \mathbf{q}(t)=\mathbf{U}^T\,\mathbf{p}(t) \quad (4)$$

"eq. (4)" can be written in the modal coordinates space. In (4) $\mathbf{U}=\lfloor \mathbf{u}^1\ \mathbf{u}^2 \cdots \mathbf{u}^N \rfloor$ is the eigenvectors matrix and $\mathbf{p}(t)$ is called modal displacement vector. By substituting (4) in (3) and after multiplying this result by $\mathbf{U}^T$, the transpose matrix of $\mathbf{U}$, the following expression is obtained:

$$\mathbf{m}\frac{d^2}{dt^2}\mathbf{p}(t)+\mathbf{c}\frac{d}{dt}\mathbf{p}(t)+\mathbf{k}\mathbf{p}(t)=\mathbf{f}(t) \quad (5)$$

where:

$$\mathbf{m} = \mathbf{U}^T \mathbf{M} \mathbf{U} \tag{6.a}$$

$$\mathbf{c} = \mathbf{U}^T \mathbf{C} \mathbf{U} \tag{6.b}$$

$$\mathbf{k} = \mathbf{U}^T \mathbf{K} \mathbf{U} \tag{6.c}$$

$$\mathbf{f}(t) = \mathbf{U}^T \mathbf{F}(t) \tag{6.d}$$

It can be verified that matrices $\mathbf{m}$ and $\mathbf{k}$ are diagonal because the eigenvectors are $\mathbf{M}$ -orthogonal. Matrix $\mathbf{c}$ is not generally diagonal but, in practice, only its diagonal terms, which can be obtained experimentally, are considered. Thus, the motion equation is a single degree of freedom equations set. Now, considering that the structure is excited by a forces set of the same frequency ω_h, but with many magnitudes and phases and assuming a response of the same form, "Eq. (5)" becomes, in the frequency domain (Javadi *et all.*,1995):

$$q_i = G_{ik}(\omega_h) F_k \tag{7}$$

where $G_{ik}(\omega_h)$ is a term of the so-called mechanical structure transfer matrix, which can be expressed as:

$$G_{ik}(\omega_h) = \sum_{r=1}^{N} \left\{ \frac{1}{m_r \omega_r^2 \left(1 - \frac{\omega_h^2}{\omega_r^2}\right) + j c_r \omega_h} u_r^i u_r^k \right\} \tag{8}$$

where N is the number of modes, u_r^i is the modal coordinate in the response position i, u_r^k is the modal coordinate in the excitation position k associated to mode r and m_r, ω_r and c_r are respectively the mass, natural frequency and damping coefficient of mode r. From "Eq. (7)" the frequency response to each harmonic can be calculated taking into account the viscous damping of each mode. In this paper was considered an average damping obtained in, Neves *et al.* (1999). Figure 5 shows the calculated accelerations as a function of the frequency for the twelve first harmonics of the magnetic forces at speed of 1250 rpm.

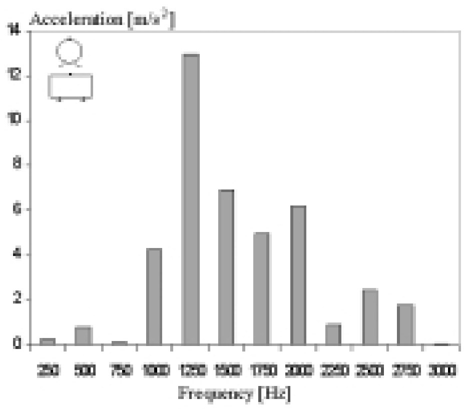

Figure 5 - Calculated acceleration as a function of frequency. The icons shows the calculation point.

The measured acceleration value for the 1250 Hz was 9.85 m/s^2 giving a difference of 32%. This difference can be reduced by making some geometric structural improvements in the model such as to put the motor covers and applying measured modal damping values obtained by Experimental Modal Analysis (Neves *et all.,* 1995). The forced deformation caused by the 5^{th} harmonic (1250 Hz) of the magnetic forces is presented in “Fig. 6”.

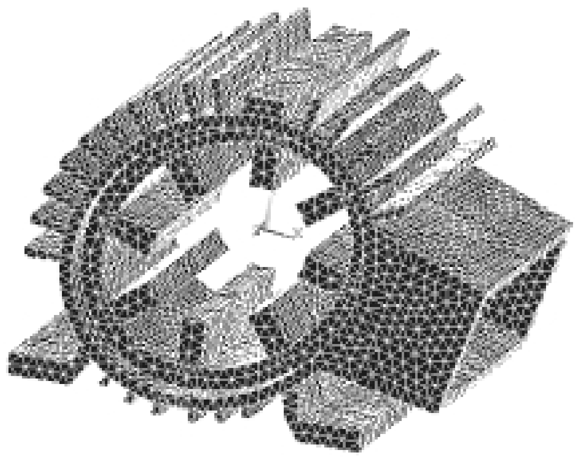

Figure 6 - Deformation caused by the 5^{th} harmonic (1250 Hz) of the magnetic forces.

6. CONCLUSIONS

A simplified approach to model magnetic forced vibrations in non-skewed electric machines is presented and applied to a 8/6 poles switched reluctance motor. The magnetic forces in the stator teeth is calculated by a method based on the Maxwell Stress Tensor. The vibrations of magnetic origin are calculated using modal superposition method. Based in the good preliminary results presented in this paper some changes in the model are being made in order to improve the results.

REFERENCES

Ewins, D. J., Modal testing theory and pratice, 1995, Research Studies Press Ltd., England.

Javadi, H., Lefèvre, Y., Clenet, S., Lajoie-Mazenc, M., 1995, Electro-magneto-mechanical characterization of the vibration of magnetic origin of electrical machines, IEEE Trans. On Magn., Vol. 31, N. 3, pp. 1892-1895.

Sadowski, N.; Lefèvre, Y.; Lajoie-Mazenc, M., Bastos, J. P. A., 1992, Sur le calcul des forces magnétiques, Journal de physique III, France, pp. 859-870.

Sadowski, N.; Lefèvre, Neves, C.G.C., Carlson, R., 1996, Finite elements coupled to electrical circuit equations in the simulation of the switched reluctance drives:attention to mechanical behavior, IEEE Trans. on Magn., Vol. 32, N.3, pp.1086-1089.

Neves, C.G.C.Simulação e análise do motor de relutância chaveado, 1995, Dissertação de Mestrado da Universidade Federal de Santa Catarina.

Neves, C.G.C., Carlson, R., Sadowski, N., Soeiro, N.S., 1997, Análise das vibrações de origem magnética no motor de relutância chaveado por meio de simulação e procedimentos experimentais, Proceedings of the 14th Congress of Mechanical Engineering, pp. 156.

Neves, C.G.C., Carlson, R., Sadowski, N., Bastos, J.P.A., Soeiro, N. S., 1999, Forced Vibrations Calculation in a Switched Reluctance Motor Taking into Account the Viscous Damping, International Electric Machines and Drives Conference (IEMDC), pp. 110-112.